1 Logistics

Next meeting: Tuesday April 9, 11:30 - 1pm

Expectations for next meeting:

- angle constraints on hexagon crease pattern

  Problem 11: Suppose we want to fix three consecutive creases on a hexagon at fold angles θ_1, θ_2, and θ_3. (By "fold angle" we mean flat $\Leftrightarrow$ $\theta_i = 0$.)

  (a) What combinations of $\theta_1, \theta_2, \theta_3$ are possible for a hexagon? Find exact equations to characterize this. (Hint: it may help to use `https://en.wikipedia.org/wiki/Rodrigues%27_rotation_formula`)
  One approach: given angles θ_1, θ_2, θ_3 we may calculate the distance between vertices 4 and 6 of the hexagon. This distance will be in the closed interval $[0, 2]$. For this to be a "valid" hexagon configuration though, the distance must be in the interval $[0, \sqrt{3}]$.
  What is an expression for this distance, as a function of $\theta_1, \theta_2, \theta_3$?

  Particular sub-problem:

  (c) Suppose $\theta_1 = \theta_3 = 0$ and θ_2 is arbitrary. Then one possibility is that $\theta_4 = \theta_6 = 0$ and $\theta_5 = \theta_2$. A second possibility is that crease number 5 is "flipped inward", so that $\theta_5 = -\theta_2$, and θ_4 and θ_6 are at some positive angle. What is the angle of $\theta_4 = \theta_6$ as a function of θ_2, in this case?

- [**Writing**] Write up notes for this meeting, and continue writing up relevant discussion from this week in draft of final report

- [**Final poster**] Create a shared latex file for our group's poster, using the template here: `https://sites.lsa.umich.edu/logm/resources/`

2 Tangent identity

(This is not exactly relevant for our project, just something I find mathematically interesting.)

Near $x = 0$, the tangent function has Taylor expansion

$$\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots .$$

This infinite series expansion converges when x is in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Near $x = \frac{\pi}{2}$, $\tan(x)$ has a singularity so it does not have a Taylor expansion. However, it does have a Laurent expansion (where negative powers are allowed)

$$\tan(x) = -(x - \frac{\pi}{2})^{-1} + \frac{1}{3}(x - \frac{\pi}{2}) + \frac{1}{45}(x - \frac{\pi}{2})^3 + \frac{2}{945}(x - \frac{\pi}{2})^5 + \cdots .$$

This infinite series converges when $-\frac{\pi}{2} < x < \frac{3\pi}{2}$ and $x \neq \frac{\pi}{2}$.

The tangent function even has another expression as an infinite series which is valid for *all* value x where the function is defined! This expression uses the fact that $\tan(x)$ is periodic. Since there is a singularity whenever $x = \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$, and near each singularity $x = a$ the function looks like $-(x - a)^{-1}$, we can write

$$\tan(x) = \left(\cdots - (x + \frac{1}{2}\pi)^{-1} - (x - \frac{1}{2}\pi)^{-1} - (x - \frac{3}{2}\pi)^{-1} - (x - \frac{5}{2}\pi)^{-1} + \cdots \right) + (\text{something}).$$

In fact, this “something” is just zero! The tangent function satisfies*

$$\tan(x) = \sum_{n\in\mathbb{Z}} -\left(x - \frac{(2n+1)}{2}\pi\right)^{-1}$$

(*Check: does this infinite series converge?)