1 Logistics

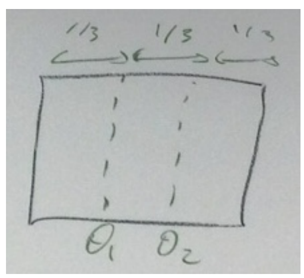
- Writing workshop coming up → let me know if you need information from me
Next meeting: Saturday February 9, 12 - 1pm.
Next next meeting: Friday February 15, 9:30 - 11am.

Expectations for next meeting:

- Focus on Problem 4: what is the space of fold configuration for 3 creases through a hexagon? Try to get a complete answer (I don't know what the answer is to this!)
- [**Writing**] Write up answer to Problem 1(b) as discussed; include some figures of the moduli space.
- [**Visualization**] Can you write code to visualize a space of fold configurations? I.e. for Problem 1 or 2, display the moduli space on one side of the screen and as you move the cursor over this space, display the fold configuration on the other side of the screen.
- Please bring back the straw polygons next week!

2 Moduli spaces

Problem 1. (a) What is the moduli space of folding configurations for two parallel creases, parallel to opposite sides, which split the sheet into 3 regions of equal area?



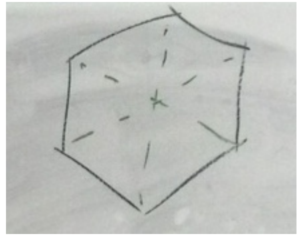
(b) What is the moduli space of folding configurations for two parallel creases, parallel to opposite sides, which are spaced arbitrarily?
(c) What distance is needed between the creases for this moduli space to look like Example 2 (the "far away" case)? What happens as the distance separating the two creases approaches 0?

Discussion: In the general case with crease spacing r_1, d, r_2, the boundary condition when the sheet intersects itself is described by the equation

$$\tan\theta_2 = \frac{r_2 \sin\theta_1}{d - r_2\cos\theta_1},$$

with certain restrictions on θ_1 and θ_2.
We looked at graphing this condition on `desmos.com` for some values of d, r_1, r_2.

Problem 4. What is the space of folding configurations of a hexagon with creases along its three axes?



3 Energy

Big question: how do materials behave in the real world?
i.e. in physics language: what configurations *minimize energy* under given constraints?

For example, an object acted on by gravity will rest at the lowest point and a spring will tend towards its equilibrium length. We are concerned not only with understanding a moduli space of all possible configurations, but also figuring out which configurations are more likely to show up due to physical considerations.

In the case of a folding sheet, a sharper fold should (usually) correspond to higher energy. Thus we can assign an *energy function* to a folding configuration Φ by, for example,

$$E(\Phi) = \sum_i |\theta_i| \ell_i$$

where $\theta_1, \theta_2, \ldots$ are the dihedral angles in the configuration Φ and $\ell_1, \ell_2, \ldots$ are the lengths of the corresponding creases ($\theta_i = 0$ indicates an unfolded crease.) More generally, we could consider the energy function

$$E_p(\Phi) = \left(\sum_i |\theta_i|^p \ell_i \right)^{1/p}.$$

Given a moduli space of fold configurations and an energy function, we would like to understand the relation between energy and other "geometric" measures of a configuration, e.g. how much space does it take up, how far away are the two farthest points, how far is it from being flat, etc.