Origami on Lattices

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Origami: a vast world of folding patterns

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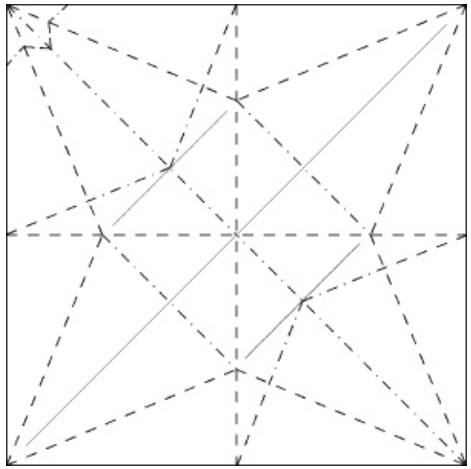
Source: http://www.1000crane.com/crane-therapy/



Source: http://www.creased.com/corners/CreasePatternCorner/creasepatternCorner.html

- What should be our strategy in understanding the behaviour of these folding patterns?
- How to analyse the geometry of the crease patterns, the restrictions on the angles?

Practical Applications: Self-Folding Origami Robots and More

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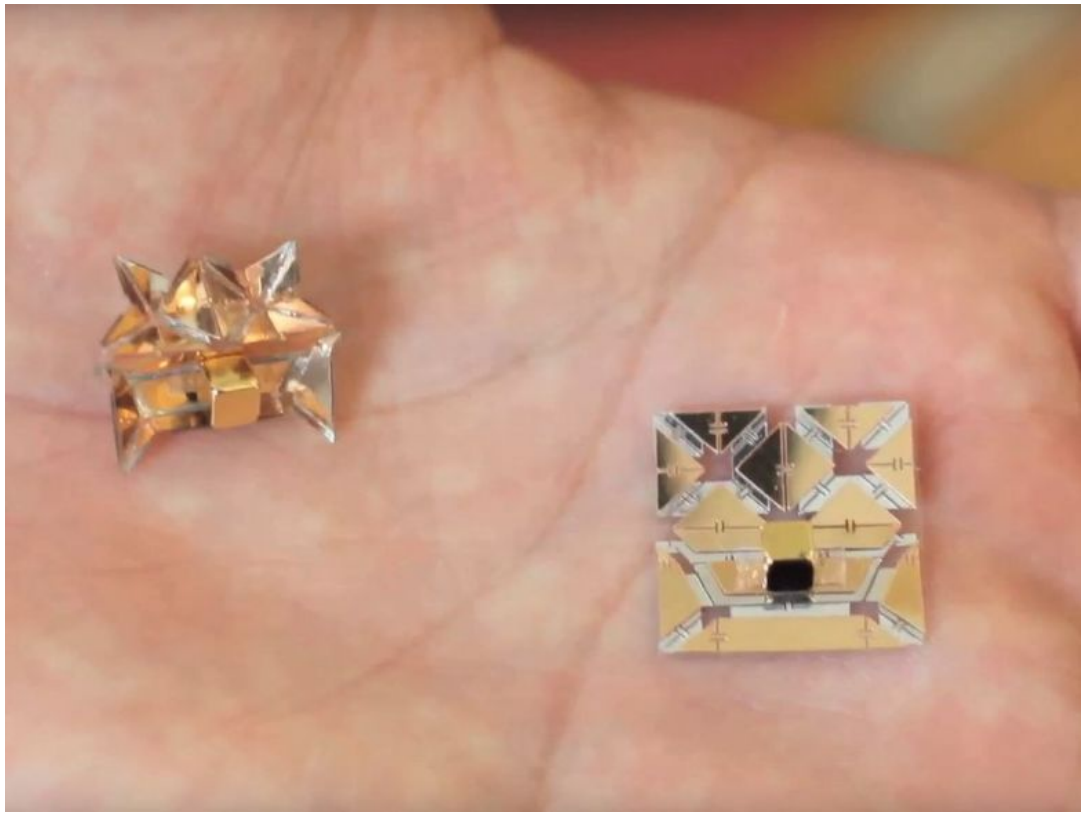

Source: https://nationalpost.com/pmn/news-pmn/canada-news-pmn/former-halifax-child-prodigy-grows-up-to-design-self-f

- What happens at smaller scales?

One Crease

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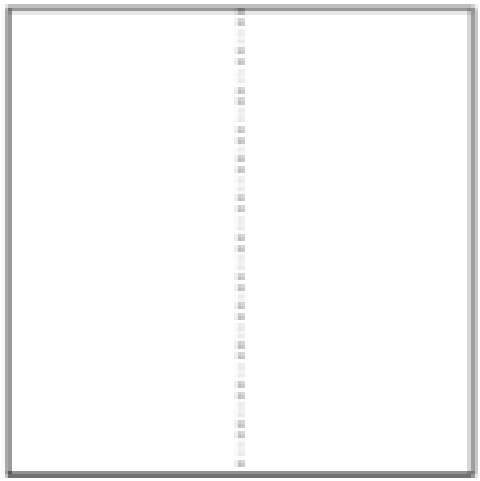

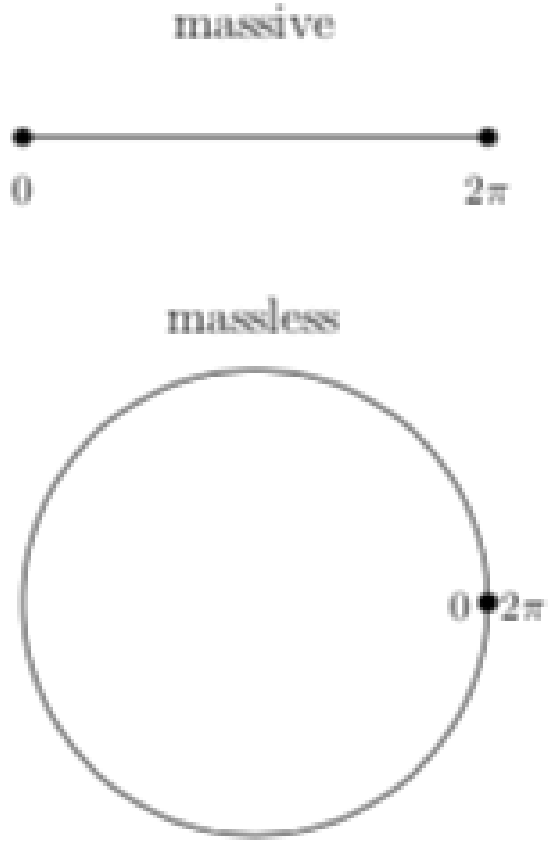

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Definition (Moduli spaces)

A *moduli space* or *parameter space* is a (topological/geometric) space which parametrizes some family of objects.

Two Parallel Creases(Massive)

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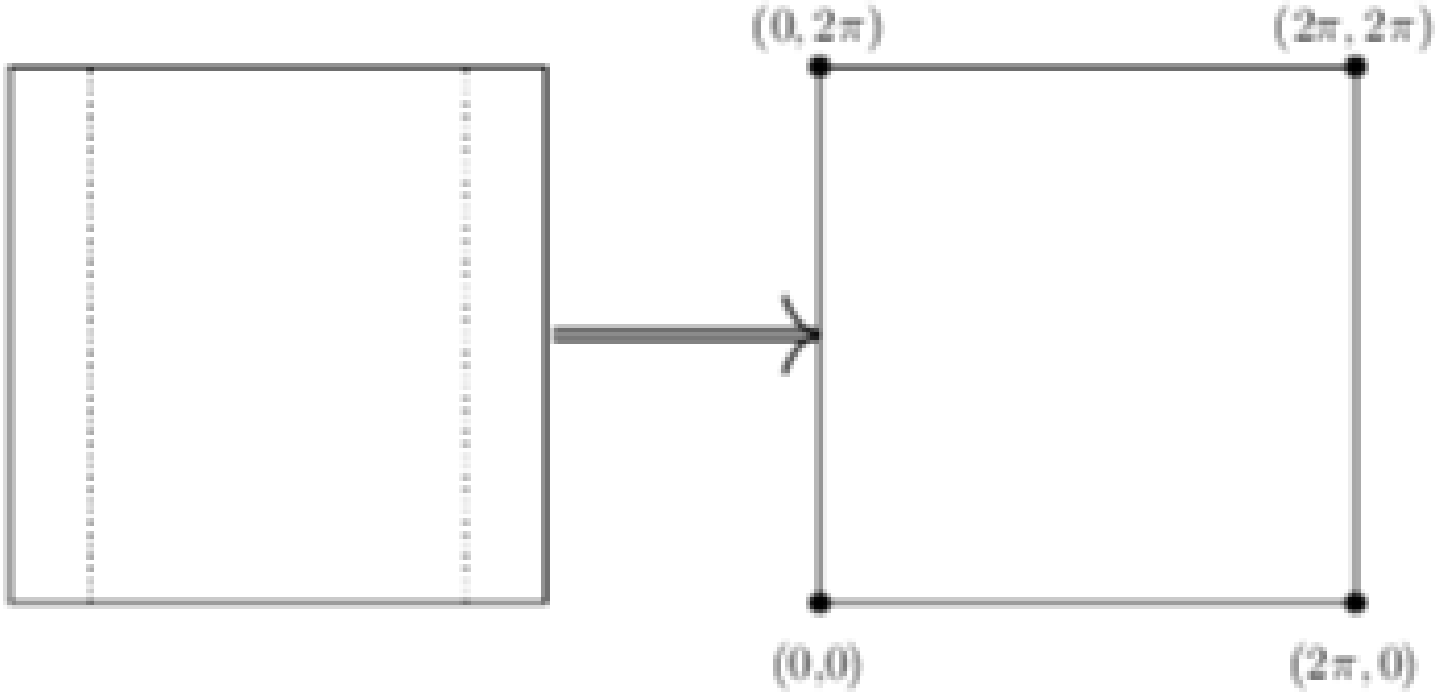

Definition (Degrees of freedom)

Degrees of freedom represents the number of independent variables that affect the space of possible configurations. Intuitively, this corresponds to the number of directions in which independent motion can occur.

Two Parallel Creases(Massless):Torus

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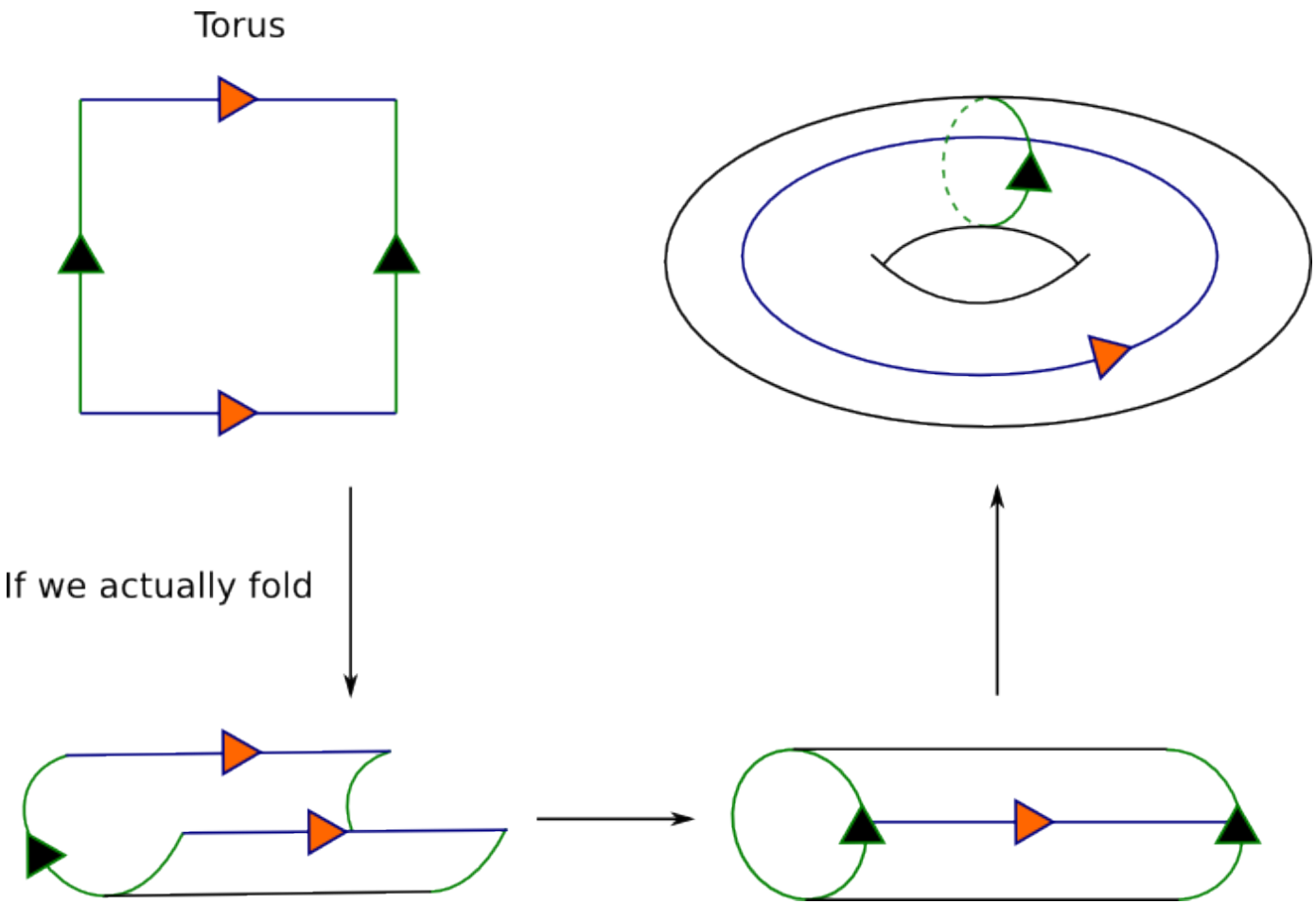

Source: http://pi.math.cornell.edu/~mec/Winter2009/Victor/part1.htm

Two Parallel Creases

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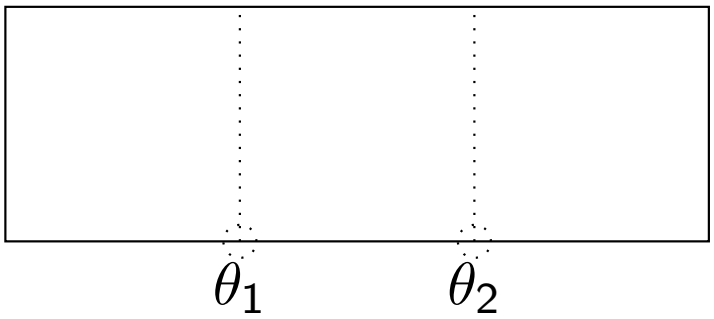
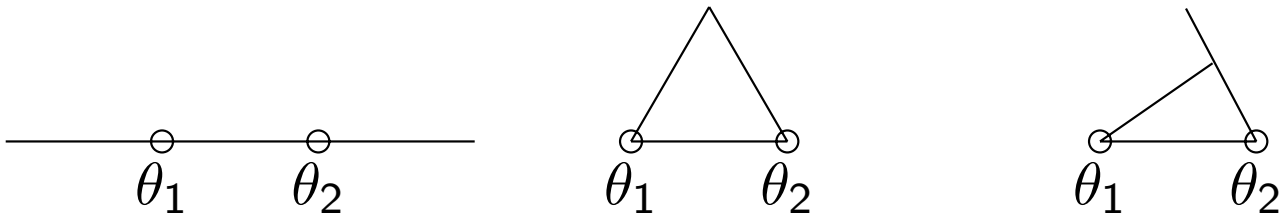
- Evenly Distributed Space.





- $\theta_1 = \pi$ - $2 * \theta_2$

Two Parallel Creases

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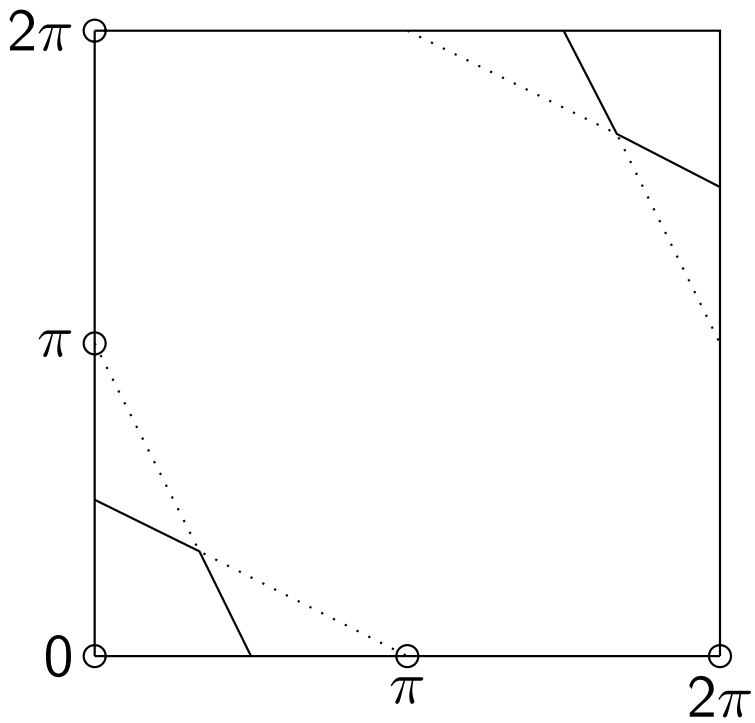
- Massive moduli space
- $\theta_1 = \pi$ - $2 * \theta_2$



Two Parallel Creases

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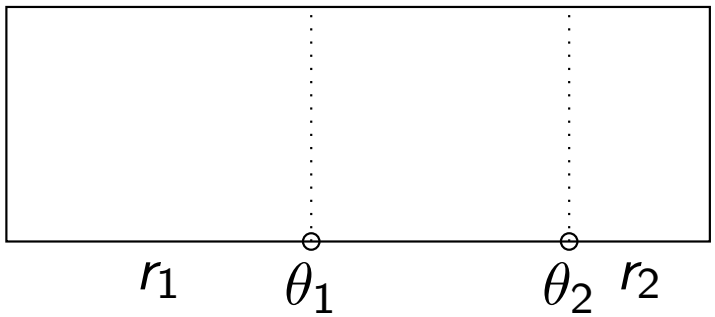
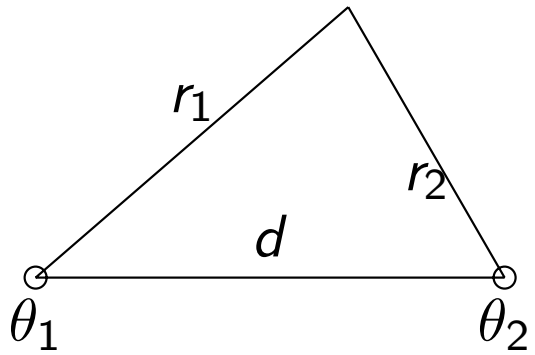
- Arbitrary Lengths
-





$$\tan\theta_1 = \frac{-r_1 \sin\theta_2}{d - r_1 \cos\theta_2} \tag{1}$$

Two Perpendicular Creases

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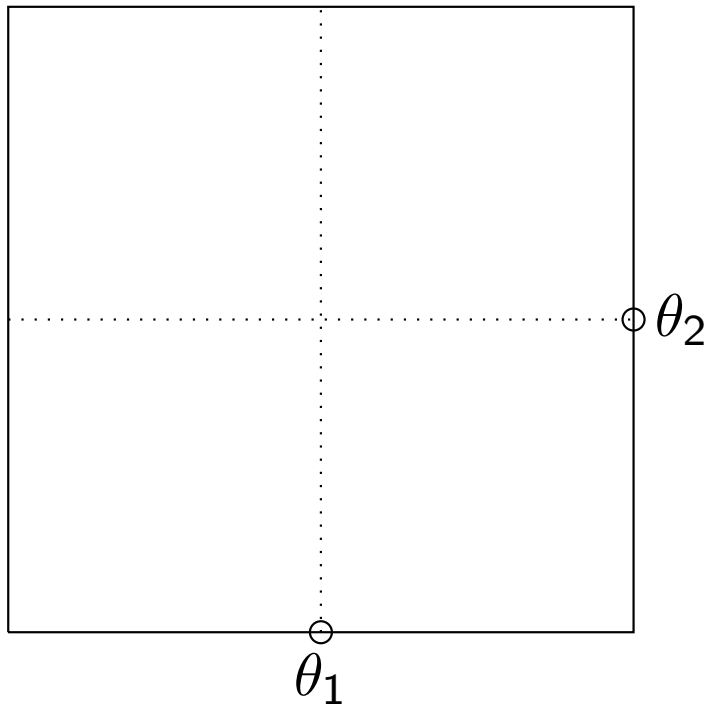
- Folding configurations for two perpendicular creases.



Two Perpendicular Creases

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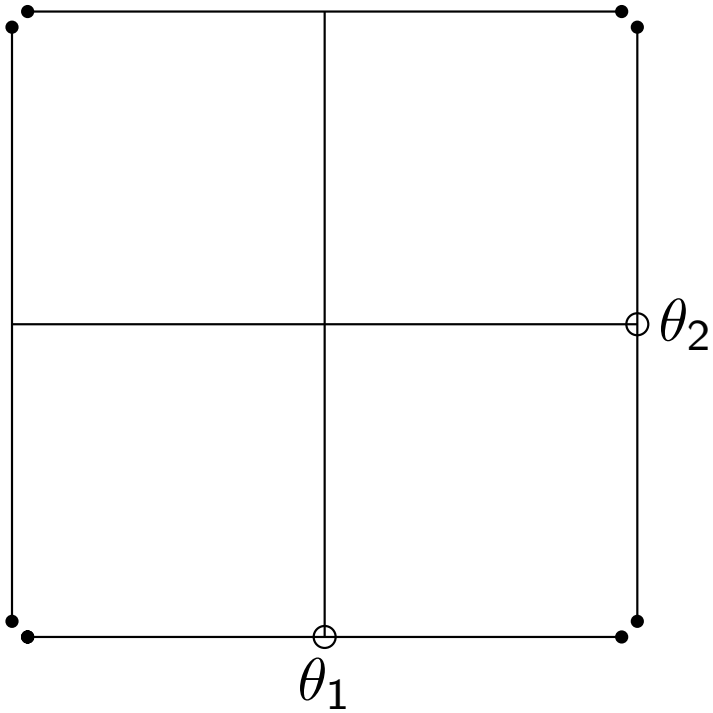
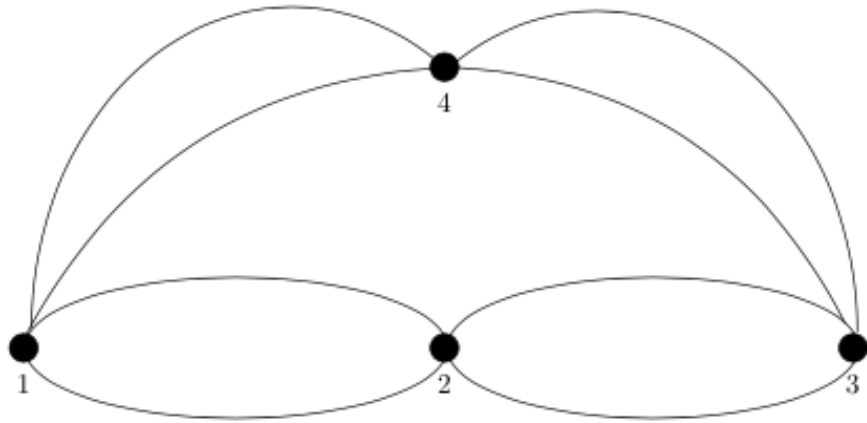
- Moduli space for folding configurations for two perpendicular creases.





Tackling the Massless Hexagon

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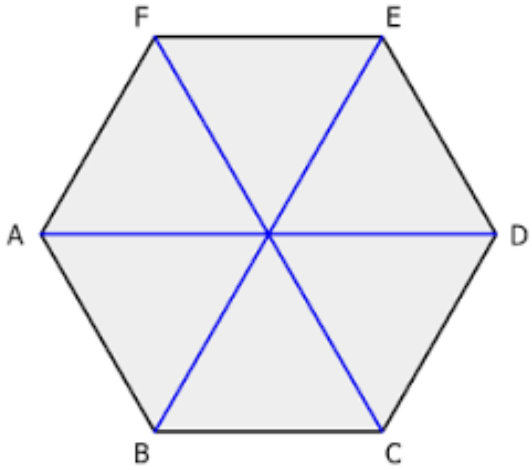

Different "coordinate" options

- Fold angles $\theta_1, ..., \theta_6 \in \mathbb{R}/2\pi\mathbb{Z}$
- Crease vectors $u_1, ..., u_6 \in \mathbb{R}^3$
  Note: these vectors form a valid configuration iff the following constraints are satisfied $\|u_i\| = 1$ for $i = 1, ..., 6$
  $\|u_i - u_{i+1}\| = 1$ for $i = 1, ..., 6$
- Estimating degrees of freedom
  (degrees of freedom) = (total dimension) - (constraints)

Future Goals - Short Term and Long Term

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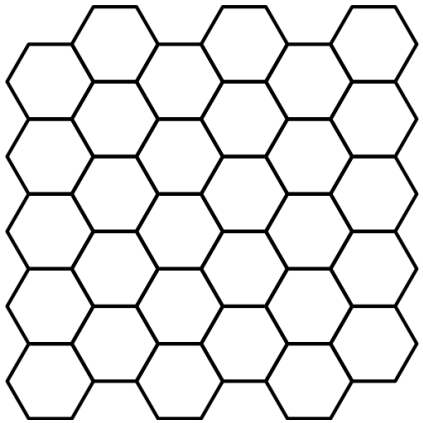

- Short Term
  - Fully characterizing the moduli space for the Hexagon (massless and massive)
  - Characterizing the moduli space for a tiling of hexagons
- Long Term
  - Characterizing the moduli space for any general lattice of folds
  - General purpose visualization tool for moduli spaces

References

[1] Bryan Gin-ge Chen, Christian D. Santangelo. Branches of Triangulated Origami Near the Unfolded State. Preprint 2018.

[2] Lab of Geometry at Michigan. LoG(M) Beamer Template. *University of Michigan Department of Mathematics*. 2018.

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